Centre Number

Student Number

SCEGGS Darlinghurst

2007

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Diagrams should be drawn in pencil
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start each question in a new booklet

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

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Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks)

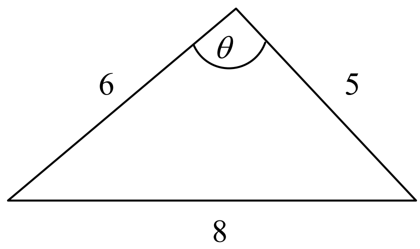
(a) Find the exact value of $\sqrt{1\frac{32}{49}}$. **1**

(b) Solve $(x-5)^2 = 16$. **2**

(c) (i) State the domain of the function $y = \dfrac{3}{2-x}$. **1**

(ii) Sketch the graph of $y = \dfrac{3}{2-x}$ showing all important features. **2**

(d) Find the value of θ in the diagram. Give your answer correct to the nearest minute. **2**



NOT TO SCALE

(e) Express $3\sqrt{5} + \sqrt{20}$ in the form $\sqrt{a}$. **2**

(f) Over 7 years, \$125 grows to \$165. Interest is compounded annually. Find the compound interest rate as a percentage per annum. Give your answer correct to one decimal place. **2**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate:

(i) $\cos\left(3x^2+1\right)$ **2**

(ii) $\dfrac{x-e^{2x}}{e^x}$ **2**

(b) Find a primitive of $\sqrt[3]{(2x+1)^2}$. **2**

(c) Evaluate $\displaystyle\int_2^3 \frac{x}{x^2-1}\,dx$ **3**

Give your answer correct to 3 significant figures.

(d) For a function $g(x)$ it is given that **3**

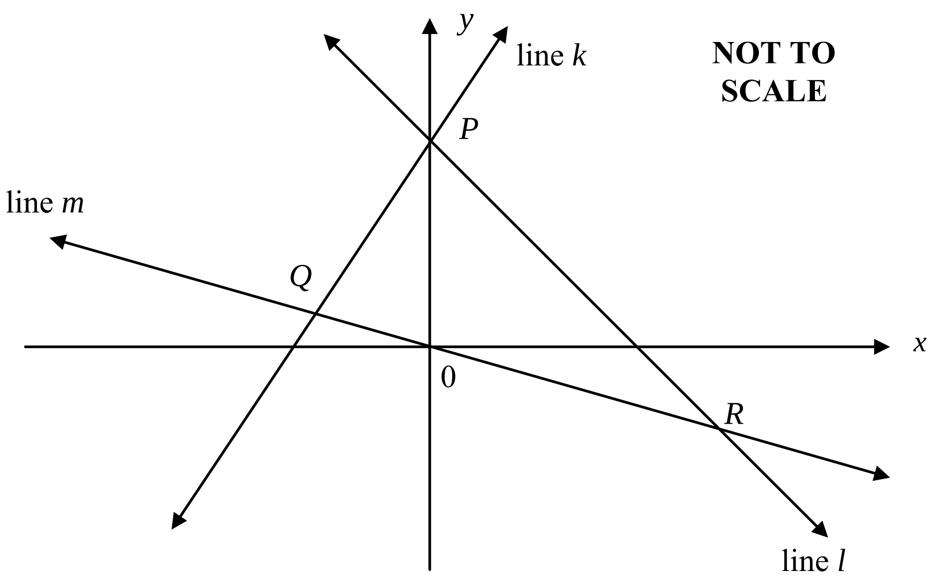
$g'(x)=3x^2-4+\dfrac{1}{x^2}$ and $g(x)=4$ when $x=1$.

Find the equation $g(x)$.

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a)



The point $Q(-2, 1)$ lies on the line *k* whose equation is $9x - 2y + 20 = 0$.
The point $R(4, -2)$ lies on the line *l* whose equation is $3x + y - 10 = 0$.

(i) By solving simultaneously, find the point *P* where *k* and *l* intersect. **2**

(ii) Find the equation of the line *m* which joins *Q* and *R*. **2**

(iii) Show that the exact perpendicular distance from *P* to line *m* is $4\sqrt{5}$ units. **2**

(iv) Hence, or otherwise, find the exact value of the area of the triangle bounded by the three lines *k*, *l* and *m*. **2**

(b) For the arithmetic progression 81, 77, 73, …

(i) show that the sum to *n* terms is given by the expression **1**

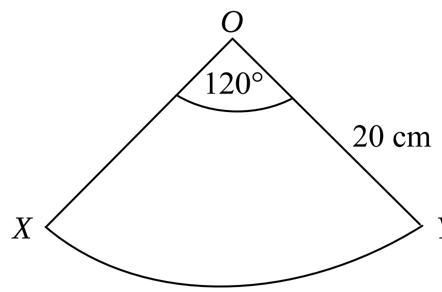
$$S_n = 83n - 2n^2$$

(ii) Find the smallest value of *n* for which the sum to *n* terms would be negative. **3**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Francesca wants to design a conical party hat for her 18th birthday party.
She folds the sector XOY so that the edges OX and OY coincide to form a cone.

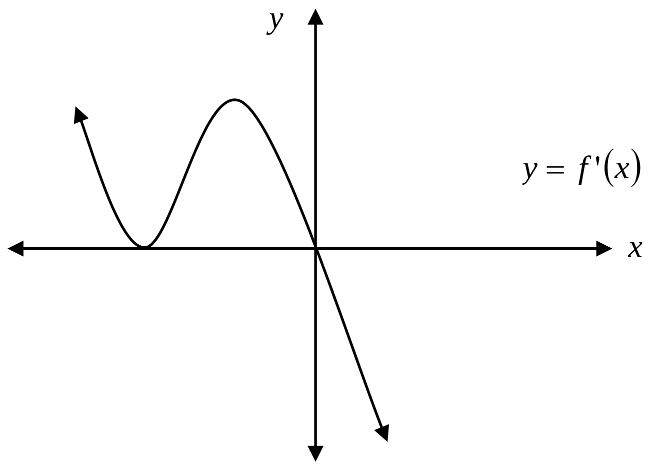


(i) Find the exact length of the arc XY. **1**

(ii) Find the radius of the base of the cone formed. **1**

(b) Find the equation of the normal to the curve $y = \sin\left(2x + \frac{\pi}{2}\right)$ at the point where $x = \frac{\pi}{4}$. **3**

(c) The diagram below shows the first derivative $f'(x)$ of a function. **3**
Copy the diagram into your answer booklet and on the same axes draw a possible curve for the function $f(x)$.



Question 4 continues on page 6

Marks

Question 4 (continued)

(d) A box contains 10 chocolates all of identical appearance.
Four have caramel centres and the other six have mint centres.
Jolene randomly selects and eats three chocolates from a box.

Find the probability that Jolene eats:

(i) three mint chocolates. **1**

(ii) exactly one caramel chocolate. **2**

(iii) at least one mint chocolate. **1**

Marks

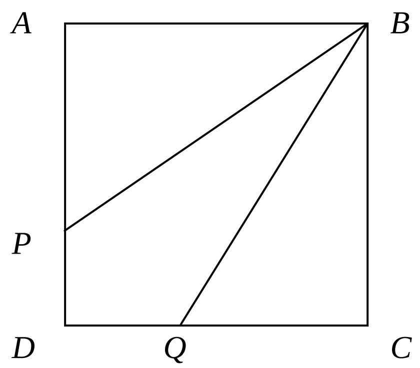
Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) A quadratic equation with roots α and β has the form **2**

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Hence, or otherwise, form a quadratic equation whose roots are $3+\sqrt{5}$ and $3-\sqrt{5}$.

(b)



NOT TO SCALE

ABCD is a square. *P* and *Q* are points on *AD* and *DC* respectively such that $PD = QD$.

Copy the diagram into your answer booklet.

(i) Prove that ΔBAP is congruent to ΔBCQ. **3**

(ii) If $\dfrac{BP}{BA} = \dfrac{3}{2}$ find the exact value of $\tan\angle APB$. **2**

(c) Luigi decides to set up a trust fund for his grand-daughter Sophia. He plans to give it to her on her 21st birthday. He invests \$150 at the beginning of each month. The money is invested at 9% per annum, compounded monthly.

The trust fund matures at the end of the month of his final investment, 21 years after his first investment. This means that Luigi makes 252 monthly investments.

(i) After 21 years, what will be the value of the first \$150 invested? **2**

(ii) By writing a geometric series for the value of all Luigi's investments, the final value of Sophia's trust fund. **3**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) Consider the function defined by $f(x) = x^3(2 - x)$.

(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Find the coordinates of any point of inflexion. **2**

(iii) Sketch the graph of $y = f(x)$ for $-1 \le x \le 2$. **3**

(iv) For the given domain $-1 \le x \le 2$ when is the curve concave up? **1**

(b) Calculate the exact volume generated when the arc of the curve **3**

$$y = 2\sec\frac{x}{2}$$

between $x = \frac{\pi}{3}$ and $x = \frac{\pi}{2}$ is rotated about the x axis.

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Solve the equation: **2**

$$9^x - 4.3^x + 3 = 0$$

(b) The point $P(x, y)$ moves such that its distance from the point $R(-1, 0)$ is always twice its distance from the point $S(2, 0)$.

(i) Show that the equation of the locus of point P is given by $x^2 - 6x + y^2 + 5 = 0$. **2**

(ii) Describe the locus geometrically. **2**

(c) For the curve $y = 2\sin x - 1$.

(i) Find the roots of the equation $2\sin x - 1 = 0$ for $0 \le x \le 2\pi$. **2**

(ii) Sketch the curve for $0 \le x \le 2\pi$ showing all important features and points of intersection with the x axis. **2**

(iii) Find the area under the curve that lies above the x axis in the given domain. **2**

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) (i) For what values of x will the geometric progression **1**

$$1+(x-2)+(x-2)^2+\ldots$$

have a limiting sum?

(ii) If this series has a limiting sum of 2, find the value of x. **2**

(b) Consider the curve $f(x)=\dfrac{e^x}{x}$.

(i) State the domain of $y=f(x)$. **1**

(ii) Show that $f'(x)=\dfrac{e^x(x-1)}{x^2}$. **1**

(iii) Find the co-ordinates of the stationary point and determine its nature. **3**

(iv) Explain why $y=f(x)$ has no x-intercepts. **1**

(v) What happens to $f(x)$ as $x\to-\infty$. **1**

(vi) Sketch the curve $y=f(x)$, showing all important features. **2**

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) Find the co-ordinates of the vertex and the focus of the parabola **2**

$$y = x^2 - 6x + 10$$

(b) (i) Use Simpson's rule with five function values to evaluate $\int_1^3 5^{2x}\, dx$ correct to three decimal places. **2**

(ii) The region bounded by the curve $y = 5^x$, the lines $x = 1$ and $x = 3$ is rotated about the x-axis. Using part (i) or otherwise, find an estimate for the volume of revolution formed. Answer correct to three decimal places. **2**

(c) Luke has made up a new game for one person that is played with two dice. He rolls both dice and if he rolls a difference of 0 or 1 he wins but if he rolls a difference of 4 or 5 he loses. Any other difference means he rolls the dice again.

(i) What is the probability that Luke will win on his first roll of the dice? **1**

(ii) Calculate the probability that a second throw is needed. **1**

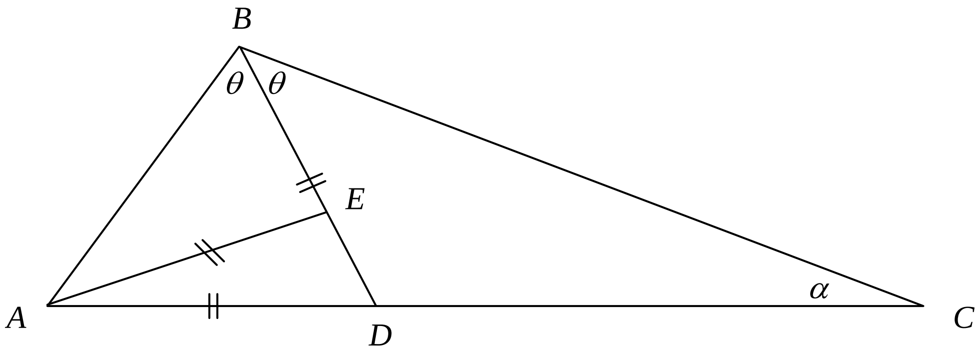
(iii) What is the probability that Luke wins on his first, second or third throw? Leave your answer unsimplified. **2**

(iv) Calculate the probability that Luke wins the game. **2**

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a)



In the diagram above, ABD and AED are isosceles triangles with $AD = BD = AE$, and BD bisects $\angle ABC$. Let $\angle ABC = \angle CBD = \theta$ and let $\angle DCB = \alpha$.

(i) Show that $\angle EAB = \alpha$, giving reasons. **3**

(ii) Hence show that $\Delta ABE \parallel \Delta CBD$. **1**

(iii) Deduce that $AE^2 = BE \times CD$. **2**

Question 10 continues on page 13

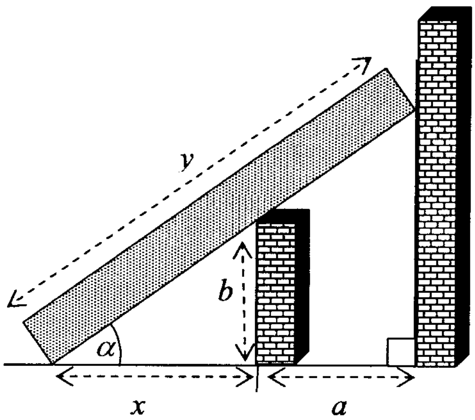
Marks

Question 10 (continued)

(b) Devendra is a motorcycle stunt bike rider in an upcoming Bollywood feature film.

He will be required to ride from a wall onto a beam, which passes over a second lower wall *b* metres high and located *a* metres from the first wall.

Let the length of the beam by *y* metres, the angle the beam makes with the horizontal be α and *x* the distance from the foot of the beam to the smaller wall.



(i) Show that $y = a\sec\alpha + b\,\mathrm{cosec}\,\alpha$. **2**

(ii) Show that **1**

$$\frac{dy}{d\alpha} = \frac{-b\cos^3\alpha + a\sin^3\alpha}{\sin^2\alpha\cos^2\alpha}$$

(iii) Hence show that if $\frac{dy}{d\alpha} = 0$ then $\tan\alpha = \sqrt[3]{\frac{b}{a}}$. **1**

(iv) Hence show that the shortest beam that can be used is given by: **2**

$$y = a\sqrt{1 + \left(\frac{b}{a}\right)^{\frac{2}{3}}} + b\sqrt{1 + \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Year 12 Trial HSC 2007 Mathematics.

Q1: Comm/2

Q1 a) $1\frac{2}{7}$ ✓

b) $(x-5)^2 = 16$

$x-5 = \pm 4$

$x = 5 \pm 4$

$x = 5-4 = 1$ $\quad$ $x = 5+4 = 9$ ✓✓

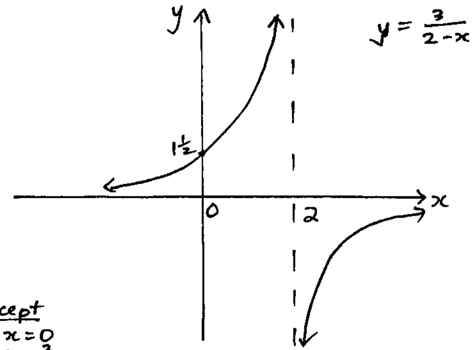
Comment: remember $\sqrt{16} = \pm 4$ so there will be two solutions

c) i) $y = \frac{3}{2-x}$

domain: all real x except $x = 2$. ✓

Comment: you must include "all real x"....

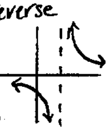
ii)



y-intercept when $x=0$, $y = \frac{3}{2}$

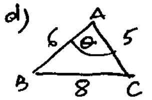
Comment: Must show
- asymptote
- y-intercept
- correct shape

- Not well done. Most had the sketch in reverse
- if you find the y-int it is obvious that this cannot be correct.



(Comm 2)

d)



$\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos\theta = \frac{5^2+6^2-8^2}{2\times5\times6}$

$\cos\theta = -0.05$ ✓

$\theta = 92^\circ 52'$ (to nearest minute.) ✓

Comment: Learn your formula

e) $3\sqrt{5}+\sqrt{20}$

$= 3\sqrt{5}+2\sqrt{5}$ ✓

$= 5\sqrt{5}$

$= \sqrt{25\times5}$ ✓

$= \sqrt{125}$

Comment: Most were fine with $5\sqrt{5}$ but many did not write in the form $\sqrt{a}$

f) $A = 165$, $P = 125$, $n = 7$, $r = ?$

Using compound interest formula.

$A = P(1+\frac{r}{100})^n$

$165 = 125(1+\frac{r}{100})^7$

Solve for r

$1.32 = (1+\frac{r}{100})^7$ ✓

$1+\frac{r}{100} = \sqrt[7]{1.32}$

$\frac{r}{100} = \sqrt[7]{1.32} - 1$

$r = (\sqrt[7]{1.32} - 1)\times 100$ ✓

$r \doteqdot 4.0$ % p.a. (1 d.p.)

Comment: Use the compound interest formula! Not series.

Q2: Calc /12

Q2 a) i) $\frac{d}{dx}(\cos(3x^2+1))$

$= -\sin(3x^2+1) \times 6x$

$= -6x\sin(3x^2+1)$ ✓✓

Comment: $\frac{d}{dx}(\cos f(x)) = -\sin f(x) \times f'(x)$

Do not attempt to use the product rule. x is only in one place.

(Calc/2)

ii) $y = \frac{x - e^{2x}}{e^x}$

$u = x - e^{2x}$ $\quad$ $v = e^x$

$u' = 1 - 2e^{2x}$ $\quad$ $v' = e^x$ ✓

Using the Quotient rule

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{e^x(1-2e^{2x}) - e^x(x - e^{2x})}{(e^x)^2}$ ✓

$= \frac{e^x - 2e^{3x} - xe^x + e^{3x}}{e^{2x}}$

$= \frac{e^x - e^{3x} - xe^x}{e^{2x}}$

$= \frac{e^x(1 - e^{2x} - x)}{e^{2x}}$

$= \frac{1 - e^{2x} - x}{e^x}$

Comment: It is better to set your work out like this rather than trying to do the steps in your head.

You only need to show you know how to use the quotient rule to get this mark. You must include the brackets.

(Calc/2)

Simplification not necessary in this question.

b) $\int \sqrt[3]{(2x+1)^2}\,dx$

$= \int (2x+1)^{2/3}\,dx$

$= \frac{(2x+1)^{5/3}}{\frac{5}{3}\times 2} + C$

$= \frac{3}{10}(2x+1)^{5/3} + C$ ✓

must have $+C$ ✓

Comment: This part was not very well done. It is an easy question. Please learn how to use the chain rule and how to change into index form.

$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$

(Calc/2)

(c) $\int_2^3 \frac{x}{x^2-1}\,dx$

$= \frac{1}{2}\int_2^3 \frac{2x}{x^2-1}\,dx$

$= \frac{1}{2}\left[\log_e(x^2-1)\right]_2^3$ ✓

$= \frac{1}{2}\left(\log_e 8 - \log 3\right)$

$= \frac{1}{2}\log_e\left(\frac{8}{3}\right)$ ✓

$\doteqdot 0.490$ (3 s.f.) ✓

$\int \frac{f'(x)}{f(x)}\,dx = \log_e f(x) + C$

You should recognise that this is a logarithm.

Use the log.law $\log a - \log b = \log \frac{a}{b}$

or just use your calculator using the [ln] key.

Calc /3

d) $g'(x) = 3x^2 - 4 + \frac{1}{x^2}$

$= 3x^2 - 4 + x^{-2}$

$g(x) = \int (3x^2 - 4 + x^{-2})\,dx$

$= \frac{3x^3}{3} - 4x + \frac{x^{-1}}{-1} + C$

$= x^3 - 4x - \frac{1}{x} + C$ ✓

(when $x=1$, $g(x) = 4$)

$4 = 1 - 4 - 1 + C$

$4 = -4 + C$

$\therefore C = 8$ ✓

Equation of $g(x)$

$g(x) = x^3 - 4x - \frac{1}{x} + 8$ ✓

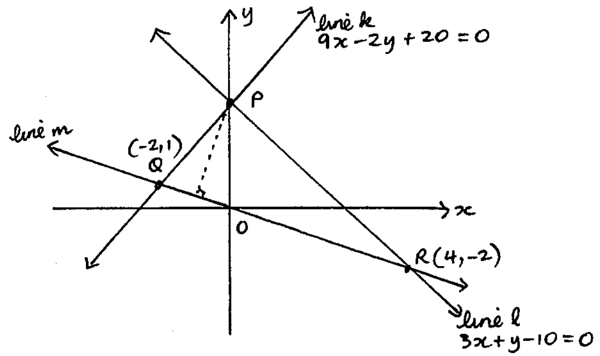
You will have more success if you write all terms in index form before attempting to integrate.

Don't forget to include +C in this question.

This is a standard question. Make sure you practise this type of integration.

Calc /3

Q3 (a)



Q3: Reas /3

i) $9x - 2y + 20 = 0$ ①

$3x + y - 10 = 0$ ②

② × 2 $6x + 2y - 20 = 0$ ③

$9x - 2y + 20 = 0$ ①

Add ③ & ①

$15x = 0$

$x = 0$ ✓

Subst. into ①

$-2y + 20 = 0$

$2y = 20$

$y = 10$ ✓

$\therefore$ P has coordinates $(0, 10)$

Well Done.

ii) $Q(-2,1)$ $R(4,-2)$

$m_{QR} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-1}{4+2}$

$= \frac{-3}{6}$

$= -\frac{1}{2}$ ✓

Equation of QR

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{1}{2}(x+2)$

$2y - 2 = -x - 2$

$x + 2y = 0$ ✓

Well Done

Q3 a) cont'.

iii) $P(0,10)$ line m $x+2y=0$

$$pd = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$= \frac{|0+2\times 10|}{\sqrt{1^2+2^2}}$$

$$= \frac{20}{\sqrt{5}}$$ ✓

$$= \frac{20}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{20\sqrt{5}}{5}$$

$$= 4\sqrt{5} \text{ units.}$$ ✓

Well Done.

- you must show the process of rationalising the denominator not just go from $\frac{20}{\sqrt{5}}$ to $4\sqrt{5}$
- this is necessary as $4\sqrt{5}$ is given to you

iv) $QR = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$= \sqrt{(-2-4)^2+(1+2)^2}$$

$$= \sqrt{(-6)^2 + 3^2}$$

$$= \sqrt{36+9}$$

$$= \sqrt{45}$$

$$= 3\sqrt{5}$$ ✓

$\therefore$ Area $\Delta = \frac{1}{2} \times$ base $\times$ perp. height

$$= \frac{1}{2} \times 3\sqrt{5} \times 4\sqrt{5}$$

$$= \frac{1}{2} \times 12 \times 5$$

$$= 30 \text{ units}^2$$ ✓

Well Done

Careless errors with multiplication

(b) AP 81, 77, 73, ...

$a = 81$

$d = -4$

i) $S_n = \frac{n}{2}[2a+(n-1)d]$

$$= \frac{n}{2}[162 + (n-1)\times -4]$$

$$= \frac{n}{2}[162 - 4n + 4]$$

$$= \frac{n}{2}[166 - 4n]$$

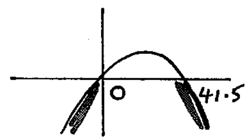
$$= 83n - 2n^2$$ ✓

Well Done

Q3 b) ii) $S_n < 0$

$$83n - 2n^2 < 0$$

$$n(83-2n) < 0$$



$n < 0$, $n > 41.5$. ✓

$n<0$ is not a valid solution ✓

$\therefore n > 41.5$

$n = 42$ ✓

- This is a quadratic inequality, you MUST do a sketch to solve this
- MUST state that $n<0$ is not a valid solution
- Answer the question asked: $n = 42$

Reas /3

Q4 a) $\theta = 120°$

$$= \frac{2\pi}{3} \text{ radians}$$

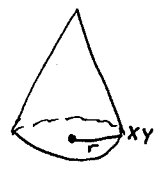
$$l = r\theta$$

$$XY = 20 \times \frac{2\pi}{3}$$

$$= \frac{40\pi}{3} \text{ cm}$$ ✓

Q4: Reas /1, comm /3, calc /3

- done very well
- leave expression as a improper fraction



the points XY join up to form the base of the hat, so the arc XY is the circumference of the base of the hat.

$$2\pi r = \frac{40\pi}{3}$$

$$r = \frac{\frac{40\pi}{3}}{2\pi}$$

$$r = \frac{20}{3} \text{ cm}$$ ✓

- this was done poorly and this was due to mainly candidates not knowing the formula for the circumference of a circle.
- there was also many silly errors

Reas 1

Q4 b)

$y = \sin(2x + \frac{\pi}{2})$

$y' = \cos(2x + \frac{\pi}{2}) \times 2$

$= 2\cos(2x + \frac{\pi}{2})$

---

when $x = \frac{\pi}{4}$

tangent $m_1 = 2\cos(2 \times \frac{\pi}{4} + \frac{\pi}{2})$

$= 2\cos(\frac{\pi}{2} + \frac{\pi}{2})$

$= 2\cos\pi$

$= -2$

normal $m_2 = \frac{1}{2}$ ✓

– done fairly well by most candidates but there are still some who need to practice working with radians

---

when $x = \frac{\pi}{4}$

$y = \sin(2 \times \frac{\pi}{4} + \frac{\pi}{2})$

$= \sin\pi$

$= 0$ $\quad (\frac{\pi}{4}, 0)$ ✓

– it was pleasing to see nearly all students know the relationship between gradients and perpendicular lines.

Equation of normal

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{1}{2}(x - \frac{\pi}{4})$

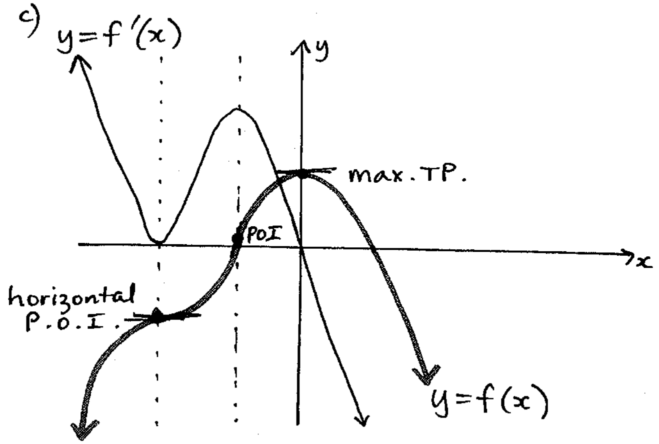
$y = \frac{1}{2}x - \frac{\pi}{8}$ ✓

$x - 2y - \frac{\pi}{4} = 0$

Calc/3

---

c)



Slope diagram.

+ 0 + 0 −

/ − / − \

Horizontal POI ✓

max TP. ✓

concavity correct ✓

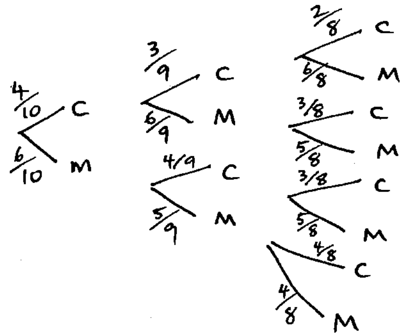
– most common mistake was sketching the derivative of the graph!

– It is really important students try to be as neat as possible with these graphs i.e. line up where the max tp's and p.o.i AND LABEL!!!

Comm/3

---

4 d)



i) $P(MMM) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$

$= \frac{1}{6}$ ✓

– Done fairly well by candidates although quite a few didn't realise there were no repeats

ii) P(exactly one C)

$= P(CMM) + P(MCM) + P(MMC)$

$= (\frac{4}{10} \times \frac{6}{9} \times \frac{5}{8}) \times 3$ ✓ ✓

$= \frac{1}{2}$

– Quite a few students didn't realise there was 3 outcomes here NOT 2 outcomes.

iii) P(at least one M)

$= 1 - P(\text{no } M)$

$= 1 - P(CCC)$

$= 1 - (\frac{4}{10} \times \frac{3}{9} \times \frac{2}{8})$

$= \frac{29}{30}$ ✓

– Done fairly well.

Q5: Reas /7
Comm /3

Q5 a) Sum of roots.

$$\alpha+\beta = 3+\sqrt{5} + 3-\sqrt{5}$$
$$= 6$$

Product of roots

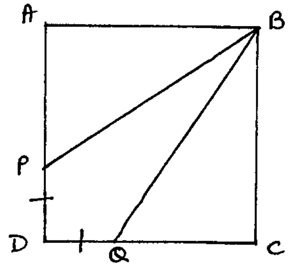
$$\alpha\beta = (3+\sqrt{5})(3-\sqrt{5})$$
$$= 9-5$$
$$= 4$$

$\therefore$ Equation (Must have $=0$)

$$x^2 - 6x + 4 = 0$$

Very poorly done, despite the fact that all the required theory was given in the question!

b)



Given
ABCD is a square
PD = QD

i) In ΔBAP and ΔBCQ

AB = BC (equal sides of a square)

$\angle BAP = \angle BCQ = 90°$ (ABCD is a square)

Since AD = DC (sides in a square)
and PD = QD (given)
$\therefore$ AP = AD − PD
QC = DC − QD
$\therefore$ AP = QC

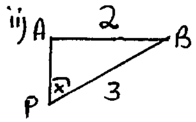
This particular reason was not well explained.

$\therefore \Delta BAP \equiv \Delta BCQ$ (SAS)

Many forgot this, or gave the incorrect test.

Comm/3

ii)



$$\frac{BP}{BA} = \frac{3}{2}$$

Find AP by Pythagoras.

$$AP^2 = 3^2 - 2^2$$
$$= 5$$
$$AP = \sqrt{5}$$

$$\therefore \tan\angle APB = \frac{2}{\sqrt{5}}$$

No one picked up on the fact that AP turned out to be longer than the side of the square! ... other than that it was well done.

Reas 2

Q5 c) P = \$150

r = 9% pa
= $\frac{9}{12}$ % per month
= 0.75% per month

n = 21 years
= 252 months

This was a standard "Superannuation"-type question that was very poorly done!

Let A be the amount the investment becomes.

The first investment becomes

i) $A_1 = 150(1.0075)^{252}$
$= \$985.93$

Note that questions about money should be answered to the nearest cent

ii) The second investment becomes

$A_2 = 150(1.0075)^{251}$

The third investment becomes

$A_3 = 150(1.0075)^{250}$

⋮

The last investment is

$A_{252} = 150(1.0075)$

The total investment

$$A = 150\left[1.0075^{252} + 1.0075^{251} + \ldots + 1.0075\right]$$

GP: $a = 1.0075$, $r = 1.0075$, $n = 252$, $S_n = \frac{a(r^n-1)}{r-1}$

Note that if you decide to make $a = 1.0075^{252}$ then $r = \frac{1}{1.0075}$

$$= 150 \times \frac{1.0075 \times (1.0075^{252} - 1)}{(1.0075 - 1)}$$

$$= \$112\,292.96$$

Reas /5

Q6: Comm /3, Calc /8

Overall, quite poorly done.

Incorrect application of the product rule to differentiate $x^3(2-x)$ caused many grief! Note that it is MUCH easier to expand first

6 a) i) $f(x) = x^3(2-x)$

$f(x) = 2x^3 - x^4$

i) $f'(x) = 6x^2 - 4x^3$

$f''(x) = 12x - 12x^2$

Stationary points $f'(x) = 0$

$6x^2 - 4x^3 = 0$

$2x^2(3-2x) = 0$

$x = 0 \qquad 3 - 2x = 0$

$2x = 3$

$x = 3/2$

---

when $x = 0$

$y = 0 \qquad (0,0)$

when $x = \frac{3}{2}$

$y = 2\times(\frac{3}{2})^3 - (\frac{3}{2})^4$

$= 1\frac{11}{16}$

$= 1.6875 \qquad (1\frac{1}{2}, 1\frac{11}{16})$ ✓

Test nature

when $x = 0$

$y'' = 0$

$\therefore$ possible horizontal point of inflexion.

Check concavity change $y'' = 12x - 12x^2$

$x=-1$	$x=0$	$x=\frac{1}{2}$
-24	0	3
$-$	0	$+$

$\therefore$ concavity changes, so there is a horizontal point of inflexion at $(0,0)$ ✓

← Many left part (i) here with the answer "(0,0) is a possible horizontal POI". While this is true it doesn't answer the question. You must check, here in part i, if it is or is not a POI

---

when $x = 1\frac{1}{2}$

$y'' = 12\times 1\frac{1}{2} - 12\times(1\frac{1}{2})^2$

$= -9$

$y'' < 0$ concave down ✓

$\therefore$ A maximum turning point at $(1\frac{1}{2}, 1\frac{11}{16})$

Also, when determining the nature of an S.P or checking concavity changes you must substitute actual values, simply writing + or − is not enough.

Calc /3

6 a) ii) Points of inflexion $f''(x) = 0$

$12x - 12x^2 = 0$

$12x(1-x) = 0$

$x = 0 \qquad x = 1$

Horizontal point of inflexion at $(0,0)$ from part (i) ✓

- Many forgot to check for a change in concavity
- Note that the number you choose to substitute here → $x = ?$ | $x=1$ has to be between 0 & 1

when $x = 1$

$y = 2 - 1$

$= 1 \qquad (1,1)$

check concavity change $y'' = 12x - 12x^2$

$x=\frac{1}{2}$	$x=1$	$x=1\frac{1}{2}$
3	0	-9 ← must have values
$+$	0	$-$

Since concavity changes, there is a point of inflexion at $(1,1)$ ✓

Calc/2

---

iii) Endpoints

$x = -1$

$y = -2 - 1$

$= -3$

$(-1,-3)$

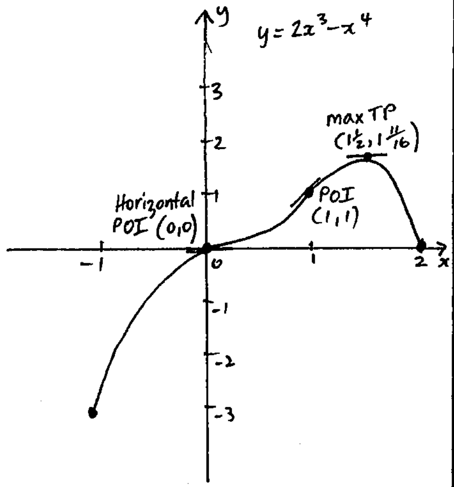
$x = 2$

$y = 2\times 8 - 2^4$

$= 16 - 16$

$= 0$

$(2,0)$



Endpoints ✓ (forgotten by many)

Stat. pts ✓

shape correct ✓

↳ Generally POIs do not have a 'kink' in them!

Comm/3

---

iv) The curve is concave up

$0 < x < 1$ ✓

This was an easy question that was very poorly done!

(13)

6 b) Volume about the x axis

$V = \pi \int_a^b y^2\,dx$

$= \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(2\sec\frac{x}{2}\right)^2 dx$

$= \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(4\sec^2\frac{x}{2}\right) dx$ ✓

$= 4\pi \left[\frac{1}{\frac{1}{2}}\tan\frac{x}{2}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= 4\pi \left[2\tan\frac{x}{2}\right]_{\pi/3}^{\pi/2}$ ✓

$= 8\pi \left\{\tan\frac{\frac{\pi}{2}}{2} - \tan\frac{\frac{\pi}{3}}{2}\right\}$

$= 8\pi \left\{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}\right\}$

$= 8\pi \left\{1 - \frac{1}{\sqrt{3}}\right\}\ u^3$ ✓

Not particularly well done

* Many incorrectly squared $(2\sec\frac{x}{2})^2$ to $4\sec^2 x/4$ which meant they eventually could not find a neat exact form of the volume.

* Many forgot to divide by the ½

* Note $\frac{\pi/2}{2} = \pi/4$, not $\frac{\pi}{2} \times 2 = \pi$

Calc/3

Q7 a) $9^x - 4.3^x + 3 = 0$

$(3^x)^2 - 4.3^x + 3 = 0$

Let $m = 3^x$

$m^2 - 4m + 3 = 0$

$(m-3)(m-1) = 0$ ✓

$m = 3$	$m = 1$
$3^x = 3$	$3^x = 1$
$x = 1$	$x = 0$ ✓

Q7: Reas/2, Comm/4, Calc/2

This question was really well done. Good work :)

---

b) $P(x,y)$ $R(-1,0)$ $S(2,0)$

i) $PR = 2PS$

$PR^2 = 4PS^2$

$(x+1)^2 + (y-0)^2 = 4\left[(x-2)^2 + (y-0)^2\right]$ ✓

$x^2 + 2x + 1 + y^2 = 4x^2 - 16x + 16 + 4y^2$

$3x^2 - 18x + 3y^2 + 15 = 0$ ✓

$x^2 - 6x + y^2 + 5 = 0$

Read the sentence carefully. It says that $PR = 2PS$.

Don't forget to square the 2, and don't try to fudge the answer.

Reas/2

ii) $x^2 - 6x + 9 + y^2 = -5 + 9$
complete the square

$(x-3)^2 + y^2 = 4$ ✓

∴ the locus is a circle with centre $(3,0)$ and radius 2. ✓

When completing the square, don't forget to add to both sides.

Comm/2

---

c) i) $2\sin x - 1 = 0$ for $0 \le x \le 2\pi$

$\sin x = \frac{1}{2}$

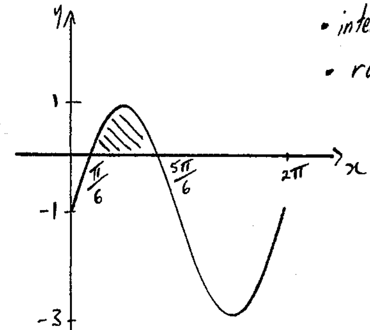
sin is positive in Q1 & Q2

$\therefore x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ ✓ both. ✓

Well done.
Use your calculator to check the angle so that you have it correct.

ii)



must show
- intercepts & domain ✓
- range ✓

Range is $-3 \le y \le 1$

Period is 2π.

If you got this wrong, practise as many trig graphs as you can. This is a standard question.

comm/2

iii)

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (2\sin x - 1)\, dx$$

$$= \Big[2\cos x - x \Big]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \checkmark$$

$$= \left\{-2\cos\tfrac{5\pi}{6} - \tfrac{5\pi}{6}\right\} - \left(-2\cos\tfrac{\pi}{6} - \tfrac{\pi}{6}\right)$$

$$= \left\{-2\times -\tfrac{\sqrt{3}}{2} - \tfrac{5\pi}{6}\right\} - \left(-2\times\tfrac{\sqrt{3}}{2} - \tfrac{\pi}{6}\right)$$

$$= \sqrt{3} - \tfrac{5\pi}{6} + \sqrt{3} + \tfrac{\pi}{6}$$

$$= 2\sqrt{3} - \tfrac{5\pi}{6} + \tfrac{\pi}{6} \checkmark$$

$$= 2\sqrt{3} - \tfrac{2\pi}{3}\ u^2 \quad \text{OR} \quad = 1.37\ u^2$$

Lots of students could integrate properly but then couldn't evaluate. You must be able to evaluate exact trig. ratios in any quadrant.

$\cos\frac{\pi}{6}$ (quad 1) $= \frac{\sqrt{3}}{2}$

$\cos\frac{5\pi}{6}$ (quad 2) $= -\frac{\sqrt{3}}{2}$

(Calc 2)

Q8

a) i) limiting sum when $-1 < r < 1$

since $r = x - 2$

$$-1 < x - 2 < 1$$

$$1 < x < 3 \checkmark$$

Q8: Reas /3, Comm /4, Calc /4

- many students know $|r| < 1$ but didn't know how to solve $-1 < x-2 < 3$
- many silly errors as well

(Reas/1)

ii)

$$S_\infty = \frac{a}{1-r}$$

$$2 = \frac{1}{1-(x-2)} \checkmark$$

$$2 = \frac{1}{3-x}$$

$$6 - 2x = 1$$

$$-2x = -5$$

$$x = \frac{5}{2} \checkmark$$

- done fairly well

(Reas/2)

---

b) i) $f(x) = \dfrac{e^x}{x}$

domain: all real x where $x \neq 0$ ✓

- Many students didn't write "all real x"

ii)

$u = e^x \qquad v = x$

$u' = e^x \qquad v' = 1$

$$f'(x) = \frac{xe^x - e^x}{x^2} \checkmark$$

$$= \frac{e^x(x-1)}{x^2}$$

- done well and clearly set out

(Calc/1)

iii) Stat. pt. when $f'(x) = 0$

$$\frac{e^x(x-1)}{x^2} = 0$$

$$e^x(x-1) = 0$$

$e^x = 0$ has no solution
$x - 1 = 0$
$x = 1$ ✓

at $x = 1$, $y = e$

test nature at $(1, e)$ ✓

x	$\frac{1}{2}$	1	2
$f'(x)$	-3.3	0	$+1.8$

✓

∴ a minimum turning point at $(1, e)$ ✓

- quite a few students forgot to comment about $e^x = 0$ has no solution
- testing of the stationary point was done fairly well
- just be careful to pick points near the stat. pt. a few students picked $x = 0$ and $x = -1$ to test which is too far from $x = 1$

If they pick < 0 don't give mark

(Calc/3)

iv) $e^x \neq 0$ for all real x and $x \neq 0$ ✓

hence $\dfrac{e^x}{x} \neq 0$ for all real x

- students needed to say $e^x \neq 0$
- there was too much focus on the denominator

(Comm/1)

v) as $x \to -\infty$

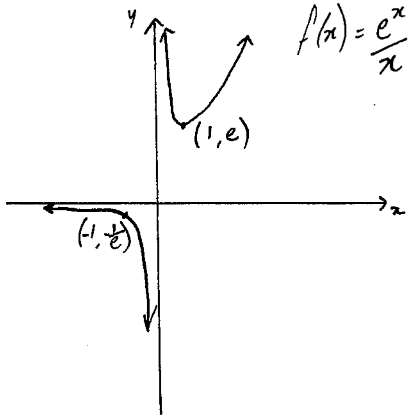
$e^x \to 0$

$\therefore \frac{e^x}{x} \to 0$ ✓

– done well

(Comm/1)

vi)



$f(x) = \frac{e^x}{x}$

Must show
- asymptote at $x = 0$
- correct shape
- stat. pt. at $(1, e)$ min.

– very poorly done. It was surprising that students who did the algebra well couldn't convert it into graphing the function correctly

(Comm/2)

Q9

a)

$y = x^2 - 6x + 10$

$x^2 - 6x = y - 10$

$x^2 - 6x + 9 = y - 1$

$(x-3)^2 = y - 1$

vertex $(3, 1)$ ✓

$f.l. = \frac{1}{4}$ $\therefore$ focus $(3, 1\frac{1}{4})$ ✓

Q9 : Reas /4
Calc /4

Not well done, you must be able to convert this to parabola form $(x-h)^2 = 4a(y-k)$ then from this get the focal length, vertex, focus.

bi) $\int_1^3 5^{2x}\,dx$ $h = \frac{3-1}{4} = \frac{1}{2}$

x	$f(x)$	factor	$f(x) \times$ factor
1	25	1	25
1.5	125	4	500
2	625	2	1250
2.5	3125	4	12500
3	15625	1	15625
		total	29900

✓ or correct substitution into formula

$\int_1^3 5^{2x}\,dx \doteqdot \frac{\frac{1}{2}}{3} \times 29900$

$\doteqdot 4983.333$ ✓

Well Done
- be careful with your calculations and substitution into the formula

(Calc/2)

ii) $V = \pi \int_1^3 y^2\,dx$

$= \pi \int_1^3 5^{2x}\,dx$ ✓

$= \pi \times 4983.333$

$= 15655.603\ u^3$ ✓

Not bad!

Use your solution from part (i) and multiply by π

(Calc/2)

c)

win	lose	roll again
0, 1	4, 5	2, 3
$P = \frac{4}{9}$	$P = \frac{1}{6}$	$P = \frac{7}{18}$

i) $P(\text{win on 1st roll}) = \frac{4}{9}$ ✓

Not good
- to win you must roll a difference of 0 or 1 there are 16 ways of doing this out of 36

(Reas/1)

ii) $P(\text{second throw}) = P(\text{roll again})$

$= \frac{7}{18}$ ✓

- A second throw needed means you must roll again ie a 2 or 3 on the first roll $= \frac{6}{36} = \frac{1}{6}$

(Reas/1)

iii) $P(\text{win 1st, 2nd, 3rd}) = P(w) + P(r, w) + P(r, r, w)$ ✓

$= \frac{4}{9} + \left(\frac{7}{18} \times \frac{4}{9}\right) + \left(\frac{7}{18} \times \frac{7}{18} \times \frac{4}{9}\right)$ ✓

$= \frac{4}{9} + \frac{4}{9} \times \frac{7}{18} + \frac{4}{9} \times \left(\frac{7}{18}\right)^2$

- write out what each roll must be and write it out
- leave unsimplified

(Reas/2)

iv) $P(\text{winning}) = \frac{4}{9} + \frac{4}{9} \times \frac{7}{18} + \frac{4}{9} \times \left(\frac{7}{18}\right)^2 + \frac{4}{9} \times \left(\frac{7}{18}\right)^3 + \ldots$

Probability forms a limiting sum with

$a = \frac{4}{9}$, $r = \frac{7}{18}$ ✓

$\therefore S_\infty = \frac{\frac{4}{9}}{1 - \frac{7}{18}}$

$= \frac{8}{11}$ ✓

- following on from part (iii) you should get a series with a limiting sum.

(Reas 2)

Q10: Reas/8
Comm/3
Calc/1

Must have
- correct reasons
- show $\angle EAB = \alpha$

Q10

ai) $\angle ADE = \theta + \alpha$ (exterior $\angle$ of ΔBCD = sum of 2 opposite interior $\angle$s) ✓

$\angle AED = \theta + \alpha$ ($\angle$s opposite = sides in an isosceles Δ are equal) ✓

$\angle AED = \angle EAB + \angle ABE$ (ext. $\angle$ of ΔABE = sum opp. int $\angle$s)

$\therefore \angle EAB = \angle AED - \angle ABE$

$= \theta + \alpha - \theta$

$= \alpha$ ✓

There is more than one way to do this question. You must make sure your reasons are clear and concise. You must have a clear reason for each of the 3 steps. Learn the correct terminology.

Comm/3

Here's another way you could do it.

$\angle BDC = 180° - (\angle DBC + \angle DCB)$

$= 180° - (\theta + \alpha)$ (Angle sum $\Delta BDC = 180°$)

$\angle BDA = 180° - \angle BDC$

$= 180° - (180° - (\theta + \alpha))$ (Angle sum of a straight line is 180°)

$= 180° - 180° + \theta + \alpha$

$= \theta + \alpha$ ✓

$\angle AED = \angle EDA$ (Angles opposite equal sides in an isosceles Δ are equal.)

$= \theta + \alpha$

$\angle BEA = 180° - \angle AED$ (Angle sum of a straight line is 180°) ✓

$= 180° - (\theta + \alpha)$

$= 180° - \theta - \alpha$

$\angle EAB = 180° - (\angle BEA + \angle ABE)$ (Angle sum $\Delta ABE = 180°$)

$= 180° - (180° - \theta - \alpha + \theta)$

$= 180° - 180° + \theta + \alpha - \theta$

$= \alpha$ ✓

Q10 (a) continued

ii) $\angle ABE = \angle CBD$ (given)

$\angle BAE = \angle BCD$ (proven in (i))

$\therefore \Delta ABE ||| \Delta CBD$ (equiangular) ✓

This part can be done independent of part (i). Please use the word equiangular not the abbreviation (AA)

Reas/1

iii) $\frac{AE}{CD} = \frac{BE}{BD}$ (corr. sides in sim Δs are in the same ratio) ✓

$AE \times BD = BE \times CD$

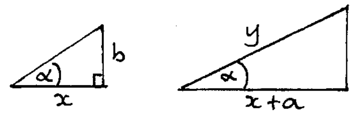
since $BD = AE$ (given) ✓

$AE^2 = BE \times CD$

You have to write the reason as well as the ratio to get this mark.

Reas/2

(b)



$\tan\alpha = \frac{b}{x}$

$x = \frac{b}{\tan\alpha}$ (1)

$\cos\alpha = \frac{x+a}{y}$

$\therefore y = \frac{x+a}{\cos\alpha}$ (2) ✓

Substitute (1) into (2)

$y = \frac{\frac{b}{\tan\alpha} + a}{\cos\alpha}$

$= \frac{b}{\tan\alpha\cos\alpha} + \frac{a}{\cos\alpha}$

$= \frac{b}{\frac{\sin\alpha}{\cos\alpha} \times \cos\alpha} + a\sec\alpha$

$= \frac{b}{\sin\alpha} + a\sec\alpha$ ✓

$= b\,\text{cosec}\,\alpha + a\sec\alpha$

Reas/2

ii) $y = a\sec\alpha + b\operatorname{cosec}\alpha$

$= \dfrac{a}{\cos\alpha} + \dfrac{b}{\sin\alpha}$

$= a(\cos\alpha)^{-1} + b(\sin\alpha)^{-1}$

using the chain rule

$\dfrac{dy}{d\alpha} = -a(\cos\alpha)^{-2} \times -\sin\alpha \quad - b(\sin\alpha)^{-2} \times \cos\alpha$

$= \dfrac{a\sin\alpha}{\cos^2\alpha} - \dfrac{b\cos\alpha}{\sin^2\alpha}$ ✓

Common denominator.

$= \dfrac{a\sin^3\alpha - b\cos^3\alpha}{\sin^2\alpha\cos^2\alpha}$

The chain Rule is.

$\dfrac{d}{dx}(f(x))^n = n f(x)^{n-1} \times f'(x)$

Note that
a is a constant
α is a variable.
If you use the quotient rule on each part
eg $\dfrac{d}{d\alpha}\left(\dfrac{a}{\cos\alpha}\right)$ $u = a$ $v = \cos\alpha$, $u' = 0$ $v' = -\sin\alpha$
Because a is a constant its derivative is 0 not 1.

Calc/1

iii) $\dfrac{a\sin^3\alpha - b\cos^3\alpha}{\sin^2\alpha\cos^2\alpha} = 0$

$a\sin^3\alpha = b\cos^3\alpha$

$\dfrac{\sin^3\alpha}{\cos^3\alpha} = \dfrac{b}{a}$

$\tan^3\alpha = \dfrac{b}{a}$ ✓

$\tan\alpha = \sqrt[3]{\dfrac{b}{a}}$

This part was done well by those who attempted it.

Reas 1

iv) min. when $\tan\alpha = \sqrt[3]{\dfrac{b}{a}}$

$y = a\sec\alpha + b\operatorname{cosec}\alpha$

since $\sec^2\alpha = \tan^2\alpha + 1$
$\sec\alpha = \sqrt{\tan^2\alpha + 1}$

also $\operatorname{cosec}^2\alpha = 1 + \cot^2\alpha$
$\operatorname{cosec}\alpha = \sqrt{1 + \cot^2\alpha}$ ✓

$\therefore y = a\sqrt{1 + \tan^2\alpha} + b\sqrt{1 + \cot^2\alpha}$

since $\tan^2\alpha = \left(\left(\sqrt[3]{\dfrac{b}{a}}\right)\right)^2 \quad \cot^2\alpha = \left(\dfrac{b}{a}\right)^{-\frac{2}{3}}$

$\therefore \tan^2\alpha = \left(\dfrac{b}{a}\right)^{\frac{2}{3}} \quad \cot^2\alpha = \left(\dfrac{a}{b}\right)^{\frac{2}{3}}$

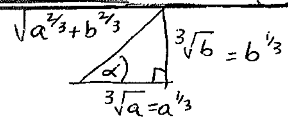
$\therefore y = a\sqrt{1 + \left(\dfrac{b}{a}\right)^{\frac{2}{3}}} + b\sqrt{1 + \left(\dfrac{a}{b}\right)^{\frac{2}{3}}}$ ✓

Congratulations if you made it this far and got this question out!

Reas 2

Here's another way to do iv).

$\tan\alpha = \dfrac{\sqrt[3]{b}}{\sqrt[3]{a}}$



$\sec\alpha = \dfrac{\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}}$

$\operatorname{cosec}\alpha = \dfrac{\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}}$

$y = a\sec\alpha + b\operatorname{cosec}\alpha$

$= a \times \dfrac{\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}} + b \times \dfrac{\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}}$

$= a^{2/3}\sqrt{a^{2/3}+b^{2/3}} + b^{2/3}\sqrt{a^{2/3}+b^{2/3}}$

$= a^{2/3}\sqrt{a^{2/3}\left(1+\left(\tfrac{b}{a}\right)^{2/3}\right)} + b^{2/3}\sqrt{b^{2/3}\left(\left(\tfrac{a}{b}\right)^{2/3}+1\right)}$

$= a^{2/3} \times a^{1/3}\sqrt{1+\left(\tfrac{b}{a}\right)^{2/3}} + b^{2/3} \times b^{1/3}\sqrt{1+\left(\tfrac{a}{b}\right)^{2/3}}$

$= a\sqrt{1+\left(\tfrac{b}{a}\right)^{2/3}} + b\sqrt{1+\left(\tfrac{a}{b}\right)^{2/3}}$

Note that $\sqrt{x^2+y^2} \neq x+y$
and $(x+y)^2 \neq x^2+y^2$
Some basic algebraic mistakes were seen in the answers to this part.